Minimizing Waiting Time of Outdoor Patients at BSMMU in Bangladesh Applying Waiting Line Theory

Md. Al-Amin Molla, M. Sharif Uddin

Abstract— Waiting lines and service systems are very important parts of the health sector. The outdoor of hospital would avoid losing their patients due to a long wait on the line. In this paper we have described some common queuing situations and presented mathematical models for analyzing waiting lines following certain assumptions those are that (1) patients come from an infinite population, (2) they follow the poisson distribution, (3) patients behaviour are treated on a service discipline “First Come First Serve” (FCFS) and do not balk or renege, (4) service time follows the exponential distribution, (5) utilization factor ρ<1 that is the average service rate is faster than the average arrival rate. In the department named department of Ophthalmology at outdoor of Bangabandhu Sheikh Mujib Medical University (BSMMU), patients have to wait long in queue for service. Thus to minimize their average waiting time for service in the queue we used waiting line theory. The model illustrated in this paper for patients on a level with service is the multi-channel waiting line model with poisson arrival and exponential service times (M/M/K: FCFS/∞/∞). We obtained the one month (January-2014) daily patients data of the above mentioned department from the medical statistic department of BSMMU. From the observation we have the arrival rate and service rate of the above department and the number of available server for each department. Then we have calculated the utilization rate, average waiting in the queue and the average number of patients in the queue, based on the data using multi-channel waiting line model (M/M/K:FCFS/∞/∞). Finally we have minimized the average waiting time in the queue for service.

Index Terms— Queues, FCFS, M/M/K, Waiting Time, Service Time
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1 Introduction

Waiting Line Theory, also known as queuing theory, is the mathematical study of queues or waiting lines. Queues or waiting lines are familiar phenomena to all. This theory can be used to model and predict waiting times and numbet of customer arrivals. Waiting for service is a part of our life. Business of all types, restaurants, checkout counters in grocery stores, industries, schools, hospitals, cafeterias, book stores, libraries, banks, post offices, petrol pums-all have queuing problems.

2 Backgrounds

The queuing theory or waiting line theory was initially proposed by Danish telephone engineer named A. K. Erlang. In the early 1903, he took up the problem on congestion of telephone traffic. The complexity was that during busy periods, telephone operators were unable to handle the calls the moment they were made, resulting in delayed calls. A. K. Erlang directed his first efforts at finding the delay for one operator and later of the results were extended to find the delay for several operators. The field of telephone traffic was further developed by Molins in 1927 and Thornton D-Fry in 1928. It was only after World War II that this early work was extended to other general problems involving queues. His works inspired engineers, mathematicians to deal with queuing problems using probabilistic methods.

Queuing theory became a field of applied probability and many of its results have been used in operations research, computer science, telecommunication, traffic engineering and reliability theory. It should be emphasized that is a living branch of science where the experts publish a lot of papers and books.

- *Md. Al-Amin Molla, Lecturer in Mathematics, Daffodil International University, Dhaka, Bangladesh. E-mail: alamin58ju@gmail.com*
- *M. Sharif Uddin, Professor, Department of Mathematics, Jahangirnagar University, Dhaka, Bangladesh. E-mail: msharifju@yahoo.com*

3 Basic Characteristics

The basic characteristics of queuing phenomenon are

- Units arrive at regular or irregular intervals of time, at a given point called the service center. For examples, person arriving a cinema hall, ships arriving a port, patients entering the doctor’s chamber and so on. All these units are called the arrivals of customers.
- At a service center there are one or more service channels or service stations. If the service stations are empty, the arriving customers will be served immediately, if not will the arriving customers wait in line until the service is provided. Once the service has been completed the customer leaves the system.

4 Elemints of Queuing Model

Customer and server are the main elements of queuing model. Customers are normally called units. It may be a person, machine, vehicles and partis etc. Server is the system which performs the services to customers. This may be single or multi-channel. Customers are generated from a source. On arrival at the facility, they can start service immediately or wait in a queue if the facility is busy. When a facility completes a service, it automatically “pulls” a waiting customer, if any, from the queue. If the queue is empty, the facility becomes idle until a new customer arrives.

A queuing model is specified completely by six characteristics:

I. **Arrival Fashion:** It represents the arriving patterns of customer in the system. Customers don’t come at a fixed regular interval of time, their arrivals are randonm fashion, it tends to be clustered or scattered randomly. In a given time, the number of arrivals is estimated by using a discrete probability distribution (DPD), such as Poisson distribution.

II. **Departure (service) Distribution:** It represents the patterns in which the number of customer leaves the system. It may also be represented by the service time, which is the time period between successive services. It may be constant or variable but known, or random (variable with only known probability). It is independent of the inter-arrival time. It is described by the exponentially probability distribution.

III. **Service Channel:** The waiting line system may have multi service channel and single service channel. Arriving customers may form one queue and get serviced, as in a doctor’s clinic. The system may have a number of service channels, which may be arranged in parallel or in series or a complex combination of both. A queuing model is called single channel model, when the system has one server only and multi-channel model, when the system has a number of parallel channels eache with one server.

IV. **Queue Discipline (service discipline):** This represents the order in which customers are selected from a queue, is an important factor in analyzing queuing models. The most common discipline is first come, first serve (FCFS), e.g. railway statons, bank ATM, doctor’s clinic etc. Other disciplines are last come, first serve (LCFS) as in big godown and service in random order (SIRO) based on priority.

V. **System Capacity:** In the system the maximum number of customer can either be finite or infinite. In limited facilities, only a finite number of customers are allowed in the system and new arriving customers are not allowed to join the system unless the number becomes less than the fixed number.

VI. **Population:** The source from which customers are generated may be finite or infinite. A finite source limits the customer’s arriving for service.

5 KENDALL’S NOTATION

In 1953, Kendall proposed a notation for summarizing the characteristics of the queuing situation, which is written as

$$(a/b/c):(d/e/f)$$

Where

$a=$ Arrival fashion, $b=$ Service time distribution, $c=$ Number of parallel servers $(1,2,3,\ldots)$, $d=$ Queue discipline, $e=$ Maximum number of units allowed in the system, $f=$ Size of the calling source.

6 MULTI-CHANNEL WAITING LINE MODEL

$$(M/M/K : FCFC/\infty/\infty)$$

Multi-channel waiting line system treats the situation in which there are two or more servers or channel in parallel to serve the arriving customers. Every server is prepared to deliver the same type of service facility. The new arrival can join any one station as he likes without any external pressure. We assume that a single line breaks down into shorter lines in front of each server. Every server has an independent and identical exponential service time distribution with mean $1/\mu$. The mean customer’s arrival fashion is λ.

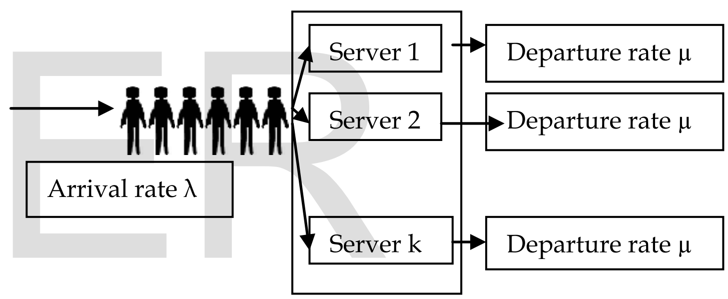

Figure-1: schematic representation of a queuing system with k parallel servers

7 CHARACTERISTICS OF THE SYSTEM

Let $n=$ number of customers in the system
$k=$ Number of parallel service channels $(k>1)$ open
$\lambda=$ Arrival rate of customers and
$\mu=$ Service rate of individual channel

1. The probability of having no customer in the system

$$p_0 = 1\Big/\left[\sum_{n=0}^{k-1}\frac{(\lambda/\mu)^n}{n!}+\frac{(\lambda/\mu)^k}{k!}\cdot\frac{k\mu}{k\mu-\lambda}\right]$$

2. Expected number of customers in the system

$$L_s=\frac{\lambda\mu(\lambda/\mu)^k}{(k-1)!(k\mu-\lambda)^2}p_0+\frac{\lambda}{\mu}$$

3. Expected number of customers waiting in the queue

$$L_q=L_s-\frac{\lambda}{\mu}$$

4. Average time a customer spends in the system

$$W_s = \frac{L_s}{\lambda}$$

5. Average waiting time of a customer in the queue

$$W_q = \frac{L_q}{\lambda}$$

6. Utilization factor: $\rho = \dfrac{\lambda}{k\mu}$

8 Waiting Tiime Analysis at BSMMU

To analyze waiting time for service in the queue, we have collected the one month (January-14), daily patients' data of the department of Ophthalmology from the medical statistics department of BSMMU.

Assumptions:

- Patients come from an infinite population,
- They follow the Poisson distribution,
- Patients behaviour are treated on a service discipline "First Come First Serve" (FCFS) and do not balk or renege,
- Service time follows the exponential distribution,
- Utilization factor ρ<1 that is the average service rate is faster than the average arrival rate
- Number of patients in the system is greater or equal to the number of service channel i.e $n \geq k$.

TABLE-1

MONTHLY PATIENT COUNTS

	Wed	Thu	Fri	Sat	Sun	Mon	Tue
1st week	48	55	0	69	0	79	99
2nd week	102	106	0	89	95	99	0
3rd week	102	79	0	81	125	105	112
4th week	118	104	0	119	118	114	135
5th week	126	156	0				

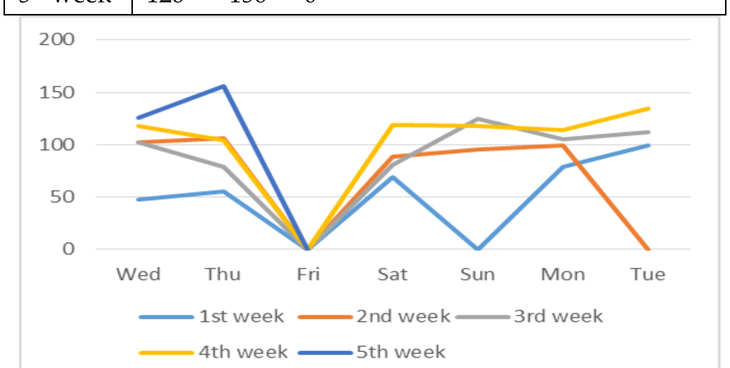

Fig. 2. One month daily patient counts

From the above figure, it is observed that, there were no patient on every Friday and Sunday of first week, Tuesday of second week, since these were holidays.

8.1 Calculation

From the observation, there are on average 23 patients coming to the Department of Ophthalmology (eye) in one hour during the time period of outdoor. So the arrival rate is $\lambda = 23$ patients per hour (pph)

Also there are on average 5 patients can take their desired service in one hour from each service channel, so the average service rate at each server is $\mu = 5$ patients per hour (pph), the number of available server in this department is $k = 5$

First at all, it is necessary to calculate the probability of having no patients in the system

$$p_0 = 1\Bigg/\left[\sum_{n=0}^{k-1}\frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^k}{k!}\cdot\frac{k\mu}{k\mu-\lambda}\right]$$

$$= 1\Bigg/\left[\sum_{n=0}^{5-1}\frac{(23/5)^n}{n!} + \frac{(23/5)^5}{5!}\cdot\frac{5\times 5}{5\times 5-23}\right]$$

$$\approx 1/[51.059 + 214.545]$$
$$\approx 1/265.604$$
$$\approx 0.004$$

[1] Average number of patients in the system

$$L_s = \frac{\lambda\mu(\lambda/\mu)^k}{(k-1)!(k\mu-\lambda)^2}p_0 + \frac{\lambda}{\mu}$$

$$= \frac{23\times 5(23/5)^5}{(5-1)!(5\times 5-23)^2}\times 0.004 + \frac{23}{5}$$

$$\approx 9.302 + 4.5$$
$$\approx 13.902$$
$$\approx 14$$

[2] Average number of patients waiting to be served

$$L_q = L_s - \frac{\lambda}{\mu} = 14 - \frac{23}{5} = 9.4 \approx 10$$

[3] Expected time a patient spends in the system

$$W_s = \frac{L_s}{\lambda} = \frac{14}{23} \approx 0.609\,hour \approx 36.54\min$$

[4] Average time a patients spends in the queue waiting for service,

$$W_q = \frac{L_q}{\lambda} = \frac{10}{23} \approx 0.435\,hour \approx 26.1\min$$

[5] Utilization factor: $\rho = \dfrac{\lambda}{k\mu} = \dfrac{23}{5\times 5} = 0.92$

Now, if we increase the number of service channel by one, i.e $k = 6$ then,

$$p_0 \approx 0.008;\ L_s \approx 6.082 \approx 7;\ L_q \approx 3.17 \approx 4;$$
$$W_s \approx 0.304\,hour \approx 18.24\min;$$
$$W_q \approx 0.174\,hour \approx 10.44\min;$$
$$\rho = 0.77$$

8.2 Evaluation

- The utilization factor ρ , is directly proportional with the mean number of patients, which means that the mean number of patients will increase as the utilization increases.
- In the department of Ophthalmology, the utilization factor is very high 0.92, in case of available server, spending time in the system and waiting time in the long queue of a patient, is reduced by half, if we increase the number of server by one.
- When the service rate is higher, the utilization will be lower, which decreases the probability of the patient going away.

8.3 Benefits

Since BSMMU is the only one medical university in Bangladesh, so this research can help this medical to ensure their QoS (Quality of Service) by anticipating, if there are many patients in the queue. It may become a model to analyze the current system and to improve the next system, as the medical can now estimate of how many patients will wait in the queue. By anticipating the huge number of patients coming to the outdoor with a view to getting services, the medical can set the proper number of service channel to ensure their desired service. This theory may be applicable for future research and could be used to develop more comples theories.

9 Conclusion

This research paper has discussed the application of multichannel queuing system at outdoor in BSMMU. Since it is a government organization and doctors are expert, usually most of the patients come to outdoor with a view to getting better treatment in the department of Ophthalmology. We have obtained that on an average a patient spends 36.54 minutes in the system and is to wait 26.1 minutes in the queue, approximately, which is indescribable sufferings for the patients, in case of available number of server and the utilization rate at outdoor is very high -0.92. So if we increase the number of server by one then a patient will spend 18.24 minutes, on an average and has to wait 10.44 minutes in the queue, which indicates that their sufferings will be appeased by half. We expect that this research can contribute not only to the betterment of the department of Ophthalmology but also other department of outdoor. One can apply this theory in restaurant, bank ATM, call center, traffic congestion etc.

As our future work, we will develop a simulation model for the medical outdoor. And then we will be able to confirm the results of the analytical model that we develop in this paper.

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